

Power Rule for Integration:

So what happens if $n = -1$?

DEFINITION: The *natural logarithmic function* is defined as

This means

Pictures:

1) Graph of $y = 1/x$

Values:

$$\ln(1/2) = \underline{\hspace{2cm}}$$

$$\ln 1 = \underline{\hspace{2cm}}$$

$$\ln 2 = \underline{\hspace{2cm}}$$

$$\ln e = \underline{\hspace{2cm}}$$

$$\ln 4 = \underline{\hspace{2cm}}$$

2) Graph of $y = \ln x$

Properties of the natural logarithm function:

1. Domain: _____ Range: _____
2. Function is continuous, increasing, and one-to-one
3. Graph is concave _____.

I. Pre-calculus

Properties of Logarithms:

1. $\ln(1) = \underline{\hspace{2cm}}$

2. $\ln(e) = \underline{\hspace{2cm}}$

2. $\ln(ab) = \underline{\hspace{4cm}}$

3. $\ln(a/b) = \underline{\hspace{4cm}}$

4. $\ln(a^n) = \underline{\hspace{4cm}}$

-examples- Expand each logarithmic expression

1. $\ln(\sqrt{3x+2})$

2. $\ln\left(\frac{xy^2}{\sqrt[3]{z}}\right)$

-examples- Condense the logarithmic expression $\ln x - 3 \ln y - \ln 2$

-example- Simplify each expression (no calculator):

1. $\ln(e^3)$

2. $\ln\left(\frac{1}{\sqrt{e}}\right)$

II. Calculus

*Derivative of Natural Logarithm Function:

-example- Find the derivative of each function.

1. $f(x) = \ln(3x + 1)$

2. $y = x \ln x$

3. $g(t) = \ln \frac{5t}{(t-4)^2}$

4. $y = \ln(\sin^2 x)$

5. Write the equation of the line tangent to $f(x) = \frac{\ln x}{x}$ when $x = e$.

*Logarithmic Differentiation: The process of using logarithms to simplify a differentiation problem.

-example- Find the derivative of $y = \frac{(x-2)^2}{\sqrt{4x+1}}$ using logarithmic differentiation.

*One final note: Because the natural logarithm function is NOT DEFINED for negative numbers, you will often encounter expressions of the form $\ln(|u|)$. You may differentiate these as though the absolute value *were not there*.

-example- $y = \ln |\cos x|$